The Distribution of Minimum of Ratios of Two Random Variables and Its Application in Analysis of Multi-hop Systems

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Abstract. *The distributions of random variables are of interest in many areas of science. In this paper, ascertaining on the importance of multi-hop transmission in contemporary wireless communications systems operating over fading channels in the presence of cochannel interference, the probability density functions (PDFs) of minimum of arbitrary number of ratios of Rayleigh, Rician, Nakagami-m, Weibull and α-μ random variables are derived. These expressions can be used to study the outage probability as an important multi-hop system performance measure. Various numerical results complement the proposed mathematical analysis.*

Keywords

Fading, ratios of random variables, Rayleigh distribution, Rician distribution, Nakagami-*m* distribution, Weibull distribution, *α-μ* distribution, multi-hop technique.

1. Introduction

In wireless communications systems, one of the serious problems is fading caused by multipath propagation. When a received signal experiences fading during transmission, signal envelope fluctuates over time [1]. There is a very wide range of statistical models for fading channels. Their accuracy and veracity depend on propagation environment and communication scenario. The most frequently applied models in the open technical literature are Rayleigh, Rician and Nakagami-*m*. Very recently, Weibull and *α-μ* distributions have also begun to receive some interest.

The distribution of the ratio of random variables is of interest in statistical analysis in biological and physical sciences, econometrics, and ranking and selection [2]. The distribution of ratios of random variables is also of interest in analyzing wireless communications systems in fading environment [3]-[7]. Namely, the random variable in nominator may present desired signal envelope while the random variable in denominator may present interference signal envelope.

In multi-hop wireless communications systems, several intermediate terminals operate as relays between the source and the destination. Namely, relays are used to carry traffic between the source terminal and destination terminal in situations when direct link is in deep fade. Since multi-hop systems are able to provide a potential for broader and more efficient coverage in bent pipe satellites and microwave links, as well as modern ad-hoc, cellular, WLAN, and hybrid wireless networks without using large power at transmitters, the performance analysis of multi-hop systems has been an important field of research in the past few years [8]-[17].

In this paper, probability density functions (PDFs) and cumulative distribution functions (CDFs) of the ratio of Rayleigh, Rician, Nakagami-*m*, Weibull and *α-μ* distributed random variables are derived. Using these expressions, PDF of minimum of arbitrary number of ratios of random variables can be easily obtained. Obtained results can be efficiently used in analysis of multi-hop systems in various fading environments.

2. System and Channel Models

As mentioned in Introduction, multi-hop transmission is a technique by which the channel from the source (*S*) to the destination (*D*) is split into several, possibly shorter, links using relays (*R*), as shown in Fig. 1.

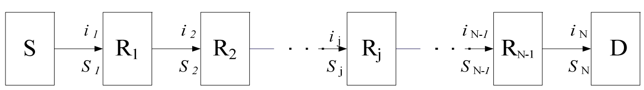

Fig. 1. The model of multi-hop system.

Signal-to-interference ratio (SIR) at the input of the *j-th* terminal can be defined as:

$$\lambda_j = \frac{S_j^2}{i_j^2}\ ,\quad j = \overline{1, N}\ , \tag{1}$$

where S_j and i_j are envelopes of desired signal and cochannel interference signal, respectively. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multipath fading envelopes. In the following equations, Ω_{1j} and Ω_{2j} represent average signal desired and interference powers, correspondingly.

- The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. The PDF expressions of S_j and i_j are [1, eq. (2.6)]

$$p_{S_j}\left(S_j\right) = \frac{2}{\Omega_{1j}} S_j\, e^{-\frac{S_j^2}{\Omega_{1j}}}, \tag{2}$$

$$p_{i_j}\left(i_j\right) = \frac{2}{\Omega_{2j}} i_j\, e^{-\frac{i_j^2}{\Omega_{2j}}}. \tag{3}$$

- The Rician distribution is used to model propagation path consisting of one strong direct LOS signal and many randomly reflected and usually weaker signals. In this case, S_j and i_j are distributed as [1, eq. (2.15)]

$$p_{S_j}\left(S_j\right) = \frac{2(K_{1j}+1)}{\Omega_{1j}} S_j\, e^{-K_{1j} - \frac{(K_{1j}+1)}{\Omega_{1j}} S_j^2} I_0\left(2S_j\sqrt{\frac{K_{1j}\,(K_{1j}+1)}{\Omega_{1j}}}\right), \tag{4}$$

$$p_{i_j}\left(i_j\right) = \frac{2(K_{2j}+1)}{\Omega_{2j}} i_j\, e^{-K_{2j} - \frac{(K_{2j}+1)}{\Omega_{2j}} i_j^2} I_0\left(2i_j\sqrt{\frac{K_{2j}\,(K_{2j}+1)}{\Omega_{2j}}}\right), \tag{5}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind and zero order. K_{1j} and K_{2j} present Rician factors defined as ratio of signal power in the dominant component over the scattered power. They describe fading severity and range from 0 (Rayleigh fading) to ∞ (no fading, i.e. constant amplitude).

- The Nakagami-*m* distribution has gained widespread application in the modeling of physical radio channels since it shows great agreement with experimentally obtained results. In Nakagami-*m* fading environment, S_j and i_j follow the distributions [18, eq. (3)]

$$p_{S_j}\left(S_j\right) = \frac{2m_{1j}^{m_{1j}}}{\Gamma\left(m_{1j}\right)\Omega_{1j}^{m_{1j}}} S_j^{2m_{1j}-1}\, e^{-\frac{m_{1j}S_j^2}{\Omega_{1j}}}, \tag{6}$$

$$p_{i_j}\left(i_j\right) = \frac{2m_{2j}^{m_{2j}}}{\Gamma\left(m_{2j}\right)\Omega_{2j}^{m_{2j}}} i_j^{2m_{2j}-1}\, e^{-\frac{m_{2j}i_j^2}{\Omega_{2j}}}, \tag{7}$$

where $\Gamma(\cdot)$ is the gamma function, m_{1j}, m_{2j} represent Nakagami parameters ($m \geq 0.5$) which determine fading severity. As parameter m increases, the fading severity decreases. The Nakagami-*m* distribution includes the one-sided Gaussian distribution ($m = 0.5$) and the Rayleigh distribution ($m = 1$) as special cases. When $m \to \infty$, the Nakagami-*m* channel converges to a non-fading channel.

- The Weibull distribution exhibits an excellent fit to experimental fading channel measurements, for both indoor [19] and outdoor [20], [21] environments. That is the reason why Weibull distribution paved its way to wireless communications applications. The PDFs of Weibull distributed random variables S_j and i_j are [22, eq. (3)]

$$p_{S_j}\left(S_j\right) = \frac{\beta_j}{\Omega_{1j}} S_j^{\beta_j-1} e^{-\frac{S_j^{\beta_j}}{\Omega_{1j}}}, \tag{8}$$

$$p_{i_j}\left(i_j\right) = \frac{\beta_j}{\Omega_{2j}} i_j^{\beta_j-1} e^{-\frac{i_j^{\beta_j}}{\Omega_{2j}}}, \tag{9}$$

where β_j is Weibull parameter expressing the fading severity. As β_j increases, the fading severity decreases, while for the special case of $\beta_j = 2$ Weibull PDF reduces to Rayleigh PDF.

- The α-μ distribution is a general fading distribution that can be used to better represent the small-scale variation of the fading signal. The PDF of α-μ distributed random variables S_j and i_j are [23, eq. (1)]

$$p_{S_j}\left(S_j\right) = \alpha \left(\frac{\mu_{1j}}{\Omega_{1j}}\right)^{\mu_{1j}} \frac{S_j^{\alpha\mu_{1j}-1}}{\Gamma\left(\mu_{1j}\right)} e^{-\frac{\mu_{1j}S_j^{\alpha}}{\Omega_{1j}}}, \tag{10}$$

$$p_{i_j}\left(i_j\right) = \alpha \left(\frac{\mu_{2j}}{\Omega_{2j}}\right)^{\mu_{2j}} \frac{i_j^{\alpha\mu_{2j}-1}}{\Gamma\left(\mu_{2j}\right)} e^{-\frac{\mu_{2j}i_j^{\alpha}}{\Omega_{2j}}}, \tag{11}$$

where α is related to the non-linearity of the environment, while μ_{1j} and μ_{2j} are associated to the number of multipath clusters. The α-μ distribution includes as special cases Nakagami-m ($\alpha = 2$) and Weibull distribution ($\mu = 1$).

3. Statistics of Ratio of Two Random Variables (SIR)

3.1 Probability Density Function

Using the following equation, we can calculate the PDF expression for SIR at the input of terminal in the j-th segment [24, 25]:

$$p_{\lambda_j}\left(\lambda_j\right) = \int_0^{\infty} |J|\, p_{S_j}\left(\sqrt{\lambda_j}\, i_j\right) p_{i_j}\left(i_j\right) di_j, \tag{12}$$

where Jacobian function can be calculated with the following equation:

$$|J| = \left|\frac{dS_j}{d\lambda_j}\right| = \frac{i_j}{2\sqrt{\lambda_j}}. \tag{13}$$

Final PDF equations of λ_j at the j-th segment can be obtained analytically in the following forms for different fading models:

- Rayleigh distribution

$$p_{\lambda_j}\left(\lambda_j\right)=\frac{\gamma_j}{\left(\gamma_j+\lambda_j\right)^{2}}, \tag{14}$$

- Rician distribution

$$p_{\lambda_j}\left(\lambda_j\right)=\sum_{g=0}^{\infty}\sum_{h=0}^{\infty}\frac{\Gamma\left(2+g+h\right)}{\left(g!\right)^2\left(h!\right)^2}e^{-K_{1j}-K_{2j}}\cdot \left(\frac{K_{1j}+1}{\gamma_j\left(K_{2j}+1\right)}\right)^{g+1}\frac{\lambda_j^g K_{1j}^g K_{2j}^h}{\left(1+\frac{\lambda_j\left(K_{1j}+1\right)}{\gamma_j\left(K_{2j}+1\right)}\right)^{2+g+h}}, \tag{15}$$

- Nakagami-m distribution [9, eq. (13)]

$$p_{\lambda_j}\left(\lambda_j\right)=\frac{\Gamma\left(m_{1j}+m_{2j}\right)m_{1j}^{m_{1j}}m_{2j}^{m_{2j}}}{\Gamma\left(m_{1j}\right)\Gamma\left(m_{2j}\right)}\cdot \frac{\lambda_j^{m_{1j}-1}\gamma_j^{m_{2j}}}{\left(\gamma_j m_{2j}+\lambda_j m_{1j}\right)^{m_{1j}+m_{2j}}}, \tag{16}$$

- Weibull distribution

$$p_{\lambda_j}\left(\lambda_j\right)=\frac{\beta_j\gamma_j\lambda_j^{\frac{\beta_j-2}{2}}}{2\left(\gamma_j+\lambda_j^{\frac{\beta_j}{2}}\right)^2}, \tag{17}$$

- α-μ distribution

$$p_{\lambda_j}\left(\lambda_j\right)=\frac{\alpha\Gamma\left(\mu_{1j}+\mu_{2j}\right)\mu_{1j}^{\mu_{1j}}\mu_{2j}^{\mu_{2j}}}{2\Gamma\left(\mu_{1j}\right)\Gamma\left(\mu_{2j}\right)}\cdot \frac{\lambda_j^{\frac{\alpha\mu_{1j}}{2}-1}\gamma_j^{\mu_{2j}}}{\left(\gamma_j\mu_{2j}+\lambda_j^{\frac{\alpha}{2}}\mu_{1j}\right)^{\mu_{1j}+\mu_{2j}}}, \tag{18}$$

where γ_j is the ratio of average powers of the desired and interference signal, i.e.

$$\gamma_j=\frac{\Omega_{1j}}{\Omega_{2j}}. \tag{19}$$

3.2 Cumulative Distribution Function

The corresponding CDF can be obtained as:

$$F_{\lambda_j}\left(\lambda_j\right)=\int_0^{\lambda_j}p_{\lambda_j}\left(x\right)dx. \tag{20}$$

Final equations for the CDF of λ_j have the following forms for different fading models:

- Rayleigh distribution

$$F_{\lambda_j}\left(\lambda_j\right)=\frac{\lambda_j}{\gamma_j}\,{}_2F_1\left(2,1,2;-\frac{\lambda_j}{\gamma_j}\right), \tag{21}$$

- Rician distribution

$$F_{\lambda_j}\left(\lambda_j\right)=\sum_{g=0}^{\infty}\sum_{h=0}^{\infty}\frac{\Gamma\left(2+g+h\right)}{\left(g!\right)^2\left(h!\right)^2\left(g+1\right)}K_{1j}^gK_{2j}^he^{-K_{1j}-K_{2j}}\cdot \left(\frac{\lambda_j\left(K_{1j}+1\right)}{\gamma_j\left(K_{2j}+1\right)}\right)^{g+1}{}_2F_1\left(2+g+h,g+1,g+2;-\frac{\lambda_j\left(K_{1j}+1\right)}{\gamma_j\left(K_{2j}+1\right)}\right), \tag{22}$$

- Nakagami-m distribution [9, eq. (17)]

$$F_{\lambda_j}\left(\lambda_j\right)=\frac{\Gamma\left(m_{1j}+m_{2j}\right)}{\Gamma\left(m_{1j}\right)\Gamma\left(m_{2j}\right)m_{1j}}\left(\frac{\lambda_j}{\gamma_j}\frac{m_{1j}}{m_{2j}}\right)^{m_{1j}}\cdot {}_2F_1\left(m_{1j}+m_{2j},m_{1j},1+m_{1j};-\frac{m_{1j}}{m_{2j}}\frac{\lambda_j}{\gamma_j}\right), \tag{23}$$

- Weibull distribution

$$F_{\lambda_j}\left(\lambda_j\right)=\frac{\lambda_j^{\frac{\beta_j}{2}}}{\gamma_j}\,{}_2F_1\left(2,1,2;-\frac{\lambda_j^{\frac{\beta_j}{2}}}{\gamma_j}\right), \tag{24}$$

- α-μ distribution

$$F_{\lambda_j}\left(\lambda_j\right)=\frac{\Gamma\left(\mu_{1j}+\mu_{2j}\right)}{\Gamma\left(\mu_{1j}\right)\Gamma\left(\mu_{2j}\right)\mu_{1j}}\left(\frac{\lambda_j^{\frac{\alpha}{2}}}{\gamma_j}\frac{\mu_{1j}}{\mu_{2j}}\right)^{\mu_{1j}}\cdot {}_2F_1\left(\mu_{1j}+\mu_{2j},\mu_{1j},1+\mu_{1j};-\frac{\mu_{1j}}{\mu_{2j}}\frac{\lambda_j^{\frac{\alpha}{2}}}{\gamma_j}\right), \tag{25}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gaussian hypergeometric function.

Analytical expressions for the PDF and CDF of ratio of independent Nakagami-m random variables are published in [8], [9]. To the best of authors' knowledge, analytical expressions for the PDF and CDF of ratio of independent Rayleigh, Rician, Weibull and α-μ random variables in the form presented in this section are not published in the open technical literature. For example, the distribution of ratio of independent Rayleigh and Rician random variables is studied in [26], but the obtained PDF and CDF expressions are not the same as (14), (15), (21) and (22). Moreover, the joint distributions of ratios of random variables for the case when random variables in nominator, as well as random variables in denominator, are correlated are of great interest in performance analysis of diversity systems in real scenarios. It is the reason why this topic received great attention in the open technical literature [4], [5], [7]. Numerical results obtained using expressions for the joint PDF and CDF for the case when correlation coefficient tends to 1 coincide with corresponding numerical results presented in this paper. PDF and CDF equations of λ_j derived in subsections 3.1 and 3.2 are more suitable for calculation since they have simpler form and converge faster (for the case of Rician distribution) in comparison with infinite-series expressions for the joint PDF and CDF of ratios of correlated random variables.

4. PDF of Minimum of Ratios of Two Random Variables (SIR) and Its Applications in System Performance Analysis

Failure of multi-hop system can occur in sections source-relay, relay-relay or relay-destination, i.e. S-R_1, R_1-R_2, …, R_{j-1}-R_j, …, R_{N-1}-D when some of the values of λ_1, λ_2, ..., λ_j, ..., λ_N fall below the predetermined threshold λ_0 required to satisfy the desired quality of service (QoS). In other words, the hop with the weakest SIR determines system performance [17]. This fact implies that it is necessary to find the distribution of minimum of SIR values $\lambda = \min(\lambda_1, \lambda_2, ..., \lambda_j, ..., \lambda_N)$. PDF of λ in the case when the source terminal communicates with the destination terminal through N-1 relays (direct link is split into N segments) can be calculated by substituting appropriate PDF and CDF expressions of λ_j into the equation

$$p_\lambda(\lambda) = \sum_{j=1}^{N} p_{\lambda_j}(\lambda) \prod_{\substack{k=1 \\ k\neq j}}^{N} \left(1 - F_{\lambda_k}(\lambda)\right). \tag{26}$$

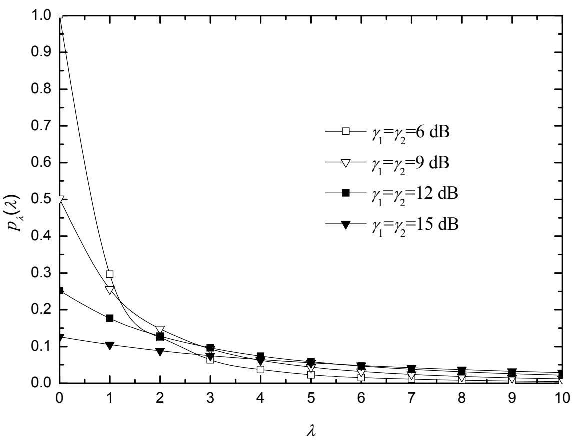

Fig. 2. PDF of minimum of SIR in dual-hop system operating over Rayleigh fading channels.

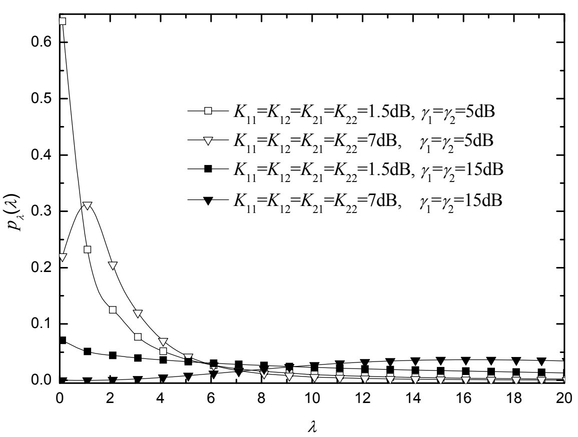

Fig. 3. PDF of minimum of SIR in dual-hop system operating over Rician fading channels.

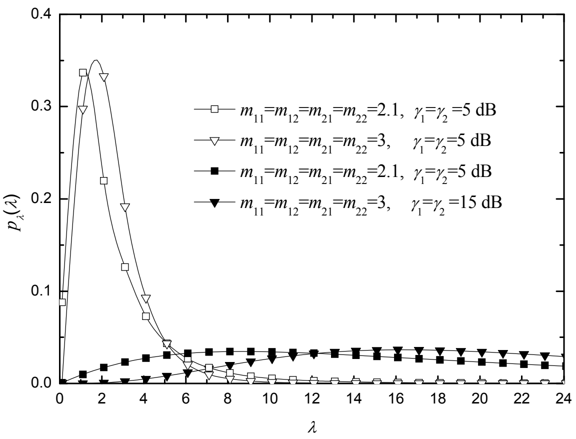

Fig. 4. PDF of minimum of SIR in dual-hop system operating over Nakagami-m fading channels.

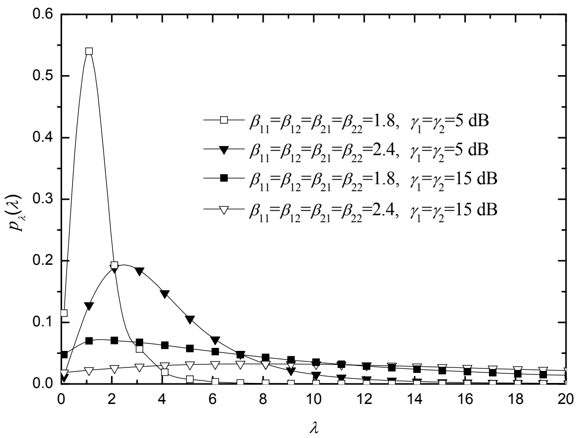

Fig. 5. PDF of minimum of SIR in dual-hop system operating over Weibull fading channels.

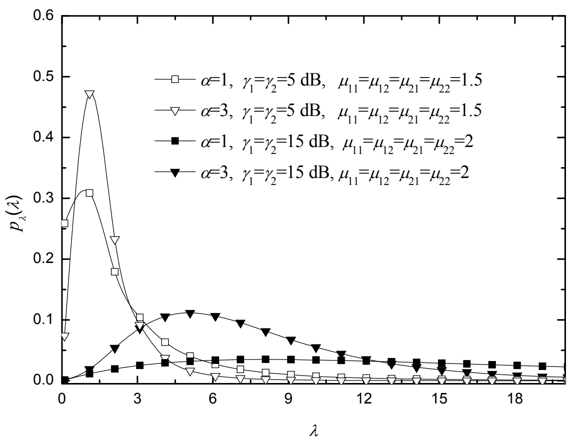

Fig. 6. PDF of minimum of SIR in dual-hop system operating over α-μ fading channels.

It must be mentioned here that $p_\lambda(\lambda)$ represents a valid PDF since it is a nonnegative function and using the well-known mathematical software packages, such as Mathematica or Matlab, it can be easily verified that $\int_0^\infty p_\lambda(\lambda) d\lambda = 1$.

With the help of program packages Mathematica 7 and Origin 8, families of curves for PDF of λ in dual-hop system for various channel parameters are plotted in Figs. 2 to 6. They reveal that increasing of ratio of average powers of desired and interference signal and decreasing of fading severity decrease the chance of taking on a lower λ resulting in better performance of the system.

The obtained analytical results for PDF can be applied to study the outage probability as an important, reliable, operable and widely accepted performance measure for wireless systems. The outage probability can be expressed as:

$$P_{out} = \int_{0}^{\lambda_{th}} p_{\lambda}(\lambda) d\lambda, \tag{27}$$

where λ_{th} is threshold, also known as a protection ratio above which the QoS is satisfactory. Using the above presented expressions, it is easy to simulate different system and channel conditions helping the designers to readjust the system's operating parameters in order to meet the QoS demands.

Numerical results for outage probability of dual-hop systems for various channel parameters are graphically presented in Figs. 7-11. It is evident that higher values of ratio of average powers of desired and interference signal lead to decreasing of the outage probability, i.e. to system performance enhancement. It is very interesting to observe that for lower values of threshold the system characterized with $\gamma_1 = \gamma_2 = 5$ dB shows better performance than the system in an environment under higher fading severity characterized with $\gamma_1 = \gamma_2 = 15$ dB. This fact confirms that fading is serious limiting factor in wireless transmission. As shown in Fig. 8 and 9, the outage probability is higher for higher values of fading severity, i.e. for lower values of Rician factor and Nakagami parameter. It is obvious that gain obtained due to increase of ratio of average powers of desired and interference signal, defined as increase of threshold for the same outage probability, is independent from fading severity. Fig. 10 and 11 depict that outage probability decreases as Weibull parameters and α parame

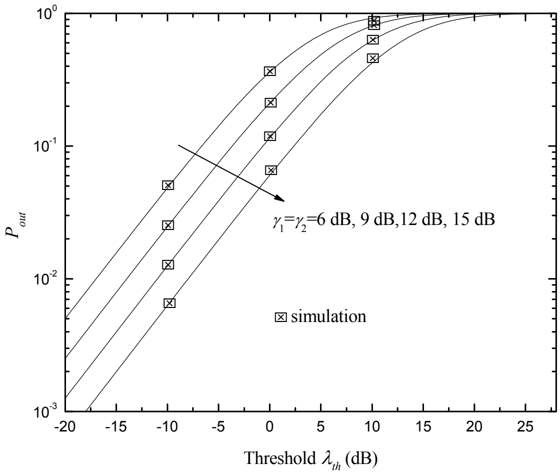

Fig. 7. Outage probability of dual-hop system in Rayleigh fading environment

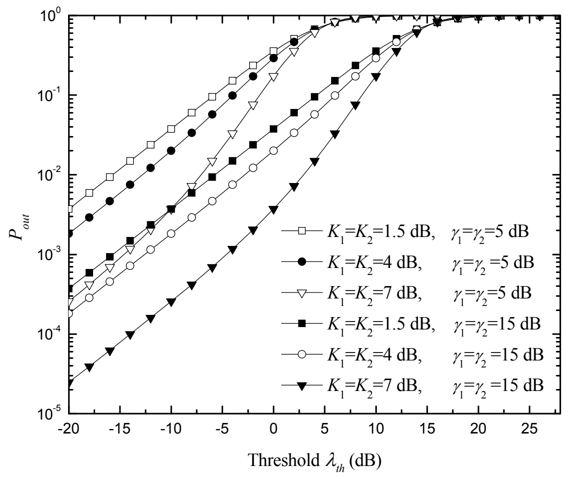

Fig. 8. Outage probability of dual-hop system in Rician fading environment.

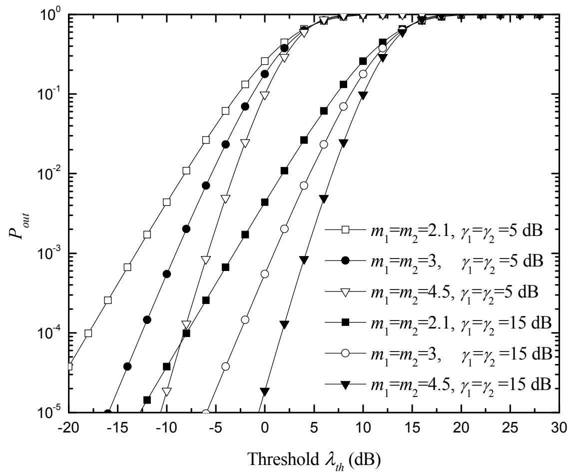

Fig. 9. Outage probability of dual-hop system in Nakagami-*m* fading environment.

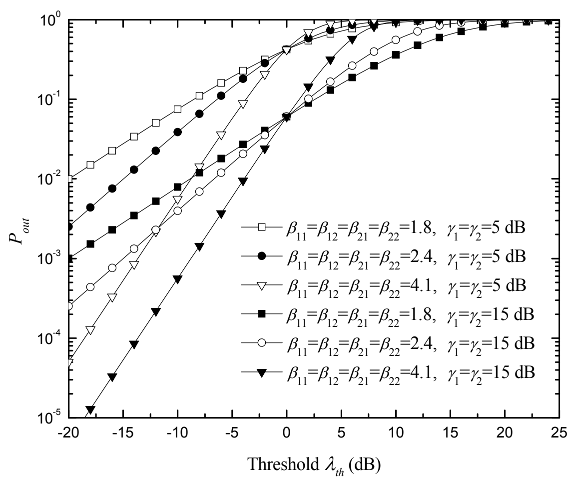

Fig. 10. Outage probability of dual-hop system in Weibull fading environment.

ter increase for $\lambda_{th} < 0$ dB. On the opposite side, for higher values of threshold, increasing of Weibull fading parameters and α parameter leads to deterioration of system performance. Also, the outage probability decreases as parameter μ increases.

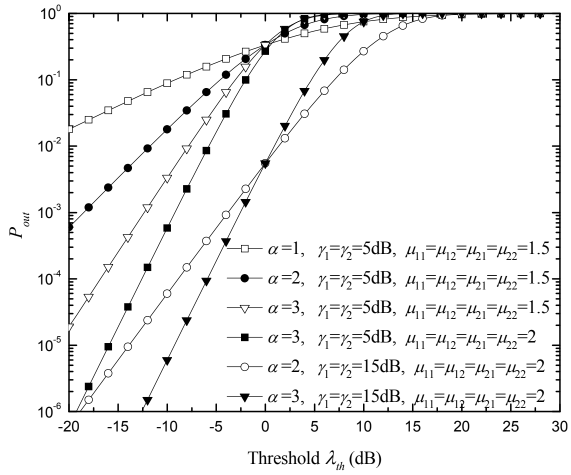

Fig. 11. Outage probability of dual-hop system in α-μ fading environment.

In Fig. 7, computer simulations for Rayleigh fading environment obtained using concept presented in [27] and Matlab 7.1 (more than 1 million samples were generated) are compared with the corresponding proposed mathematical analysis. A very good match between the theory and simulation verifies the validity and the accuracy of theoretical analysis approach presented in the paper.

5. Conclusion and Future Works

In this paper, the PDF of minimum of ratios of two random variables has been derived. Random variables in nominator, as well as random variables in denominator, are described using Rayleigh, Rician, Nakagami-*m*, Weibull and α-μ distribution enabling researchers and engineers to use our results in wide range of scenarios in many areas of science. An application of these results for the multi-hop communications as reasonable solution to achieve high data rate coverage required in future cellular wireless local area and hybrid networks, as well as to mitigate wireless channel impairments, has been described. Namely, presented results can help the designers of wireless communication systems to simulate different wireless environments and readjust system parameters in order to meet the QoS demands using the outage probability as important and widely accepted performance measure. In addition, presented PDF of minimum of ratios of random variables can be used to derive expression for moments-generating function to evaluate lower bounds for the average bit error probability, as another important system performance measure, for different modulation schemes. In order to provide enough information for the overall system design and configuration, deriving expressions for moments-generating function will be the subject of our future work.

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